Title: Distributive Property of Multiplication Over Addition -"<u>Add The Spread, Please</u>"

Brief Overview:
Add The Spread, Please! is an instructional unit consisting of three modules, Finding Factor Facts, Multiple Focus -10, and The First-Outer-Inner-Last (F.O.I. L.) Investigation. Through the Fundamental Law of Distributive, the students investigate the multiplication connection beyond repeated addition. With symbolism and representation, the students communicate how multiplication spreads across addition. While building upon conceptual and procedural knowledge of whole number computation, the students apply the distributive property as a mental math tool or strategy.

NCTM Content Standard: Number and Operations
- Understand numbers, ways of representing numbers; relationships among numbers, and number systems;
- Understand meaning of operations and how they relate to one another;
- Compute fluently and make reasonable estimations

Grades/Level:
> **Grades 4-6**
> .

Duration/Length:

> **5-day/ 55-minute Class Sessions**

Student Outcomes:

> Students will:
- recognizes equivalent representations for the same number and generate them by decomposing numbers;
- use expanded form to express whole numbers and parenthesis to evaluate the numeric expression;
- apply the place-value structure of the base-ten number system to represent whole numbers
- develop fluency with basic number combinations for multiplication and addition; use these combinations to mentally compute related problems, such as 30×50
- understand the distributive property of multiplication over addition.
- apply the distributive property as a mental math tools for computing whole numbers.

Materials and Resources:

- Centimeter graph paper
- 5 Small brown paper lunch bag
- Single colored index cards 3x5
- [Cooking] Timer
- Overhead projector
- Base-Ten Blocks (flats, rods; units)
- Cuisenaire Rods 1 set (74 per student)
- Linking or snap cubes – 1 set (100 cubes per student)
- Dominoes (Jumbo Double Fifteen Colored Dots)
- Colored Highlighters/Markers (Yellow, Orange, Light Green, Pink)
- Mathematics Made Meaningful by John Kunz (1977)
- Student Resource (SR1)EM-TAP adapted from EMAC, Ketterer, 1990
- Student Resource (SR 2) Sets of Digit cards used for factors only numbered 0-9
- Teacher Resource (TR 3) "Multiple Focus 10 "game
- Student Resource (SR 4) " Multiplicand Breakdown (place value mat)
- Teacher Resource (TR 5)"The First-Outer-Inner-Last (F.O. I. L. Method) Investigation
- Student Resource (SR 6) Summative Assessment DCPS Elementary Mathematics Standards and Curriculum Pacing Charts
- Maryland Public Schools Standards and Assessment Documents

Development/Procedures:

Lesson 1
FINDING FACTOR FACTS to express whole numbers using expanded form and using parenthesis to evaluate the numeric expression.
Pre-Assessment (Prior-Knowledge): Assess levels of basic addition and multiplication in concert. Play "Mother May I" Say My Factors. Have all students stand and when called upon provide the facts to the sum or product called by the teacher. The last child standing or after three rounds, the proficient students should be standing.

Launch: Assess students' knowledge of properties of Addition and Multiplication. Make flash cards (mock-ups) of Identity, Associative and Commutative Properties. Have students distinguish the differences and similarities of to the mystery [Distributive] property. $(a + b) + c = a = (b + c)$

$a + b + c = c + b + a$; $a + 0 = a$; $a \times 0 = 0$; $a \times 1 = a$

MYSTERY PROPERTY $a \times (b + c) = (a \times b) + (a \times c)$

Teacher Facilitation: – Present and post vocabulary (facts, factors, addends, multiplier, multiplicand, distribute, sum and product, simplify, and expanded form). Pose the question: Why must you know the facts? Presentation-Model of Writing Equivalent Expressions- as product of a factor and to factor an expression. Use EM-TAP Student Resource (SR 1) – Make an overlay to model rewriting one-digit multiplication factors into expanded form. (For example 12 x 6 = 3 x 4 +3 x 2 = 3 x (4 + 2). In cooperative groups of two, students randomly select digit cards [factors] from the brown bag. Introduce the parentheses as the symbol that ensures the order of each operation and equality. Describe what will happen if the parentheses are removed:
2 x (3 + 5) = 2 x 3 + 2 x 5 = 6 + 10 = 16.
Removed Parentheses SOLUTION 2 x 3 + 5 = 6 + 5 = 11.
Hence, 2 x (3 +5) ≠ 2 X 3 + 5 or 16 ≠ 11.
WHICH FACTOR appears in both set A and set B? Describe what is happening? Teacher needs to hear something similar to "The multiplier is being used – shared- <u>distributed among the addends.</u>" Provide guided student practice with two different colored highlighters, students highlight the multiplier and addends.

Student Application: Student independent practice/exploration. Use the EM-TAP (SR1) and Digit Cards (SR 2). Have students construct equivalent expressions using parentheses. Have students read their equivalent numeric expressions to the class.

$$\boxed{9}$$ (6 + 3); (5 + 4); (1 + 8); (2+7)

Embedded Assessment: Journal Writing Prompts
  1) Communicate and represent how to generate equivalent whole number expressions.
  2) Communicate and represent the role of the parenthesis in evaluating a numeric expression.

Re-teaching/Extension:
- Re-teaching - Start with the pre-assessment then launch. Have student state and illustrate the equivalent expressions. Have student read ready-made expressions and explain the composition of each. Reinforce with in-class peer-tutoring.
- Extension- Students may create a board game to find equivalent factor facts, using 3x5 index card and tag board.

Lesson 2

MULTIPLE FOCUS 10

Pre-Assessment: (Prior-Knowledge): Assess knowledge of place value, multiples and powers of 10 and knowledge of mental math strategies. Use Multiple Focus 10 Teacher Resource (TR 3). On 3x5 single colored index cards or on tag board, make five sets of the 16 cards represented on the Multiple Focus 10 (TR 3). Inform students that Multiple Focus 10 is a concept adapted from most memory games. All cards are placed faced-down. The students must mentally compute the numeric expression written on a selected card and try to find the matching card with the product. If a correct card pair is not located by turning over 2 cards per turn, the two cards are returned to the face-down position.

Launch: Formally reintroduce/introduce the MYSTERY property

$$a \times (b + c) = (a \times b) + (a \times c)$$

Distributive Law Solicit from students to: "Describe how to distinguish between the associative. Describe how the Commutative Property is used in the Distributive Property?" Have students assign a value to each [variable] letter and use the Order of Operation to compute total (record in journal for look-see after instruction).

Teacher Facilitation: Use [base-ten] place value. Make and

place

the Multiplicand Breakdown (SR 4) overlay on overhead. Prompts: "Describe and represent how to breakdown, simplify, expand, or rename the multiplicand." Describe the purpose of the expanded form in the computation. "Determine the multiplier and multiplicand and justify your decision" [Identify the multiplier in 3 x 95 and model the expanded form of the multiplicand, i.e., 95 = (90 + 5)]. *Guided Practice*: Monitor for demonstrated

the   understanding of the commutative property and the description of

following:
95 three times,
3 sets of 95,
three 95 times;
95 sets of 3.

Next, model procedure and flow chart [multiplication] the multiplier spreading over [addition] the addends, flow charting the procedure, [use colored overhead markers] to delineate the equality of a x (b + c) = $a \times b + a \times c$. Model the use of the Commutative property to rewrite/simplify the numeric expression a x (b + c). Point out that the Distributive property removes the parentheses. Have students use mental math focusing on multiples of ten, place value to find totals

Student Application: Exploration and Discovery Learning – Use Dominoes and the Multiplicand Breakdown (SR 3), to generate numeric expressions. Students should assign a numeric value to each [variable] letter using the dominoes. Students should expand to simplify and follow individual flow chart to find the totals. Use 2 x 34, 3 x 4, 2 x 22, or 4 x 42

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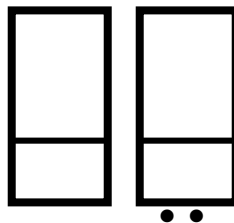
Embedded Assessment: Student applies the place-value structure of the base-ten number system to rewrite whole numbers using the distributive law, mentally computing large totals: Have students generate four expressions, rewrite, and find the totals and exchange papers for peer evaluation and teacher/group math discourse. Check problem written in Math Journal during the MULTIPLE FOCUS 10 launch.

Re-teaching/Extension
- Re-teaching: Re-teach place value multiplying a one-digit by two-digit and a two-digit by three-digit whole number. Re-teach renaming 2-digit, 3-digit numbers building tens where applicable. Re-teach multiplying by 10, 100,

1000 using the law of distributive. Encourage the meta-cognition of thinking about thinking in tens. Use Multiple Focus 10 Student Resource Pre-Assessment Lesson #2 to represent the break down into expanded form of 85. [85 = 80+5]

2	1		
Thousand	Hundreds	Tens	Ones
		8	5
		x 3	**0**
2	5	5	0

150
2400

SEE
85 x 30 = 2550 = 2400 + 100 + 50   ⟵

Suggested Samples for Extra Practice: 9**0** times 25
20**0** times 321

- Extension: Have identified students to explore simplifying and rewriting two factors and compute total. Have students write in reverse a x (b + c) as a numeric expression.

Lesson 3   **The FIRST-OUTER-INNER-LAST (F. O. I. L. METHOD)
INVESTIGATION**
Pre-assessment: Apply students knowledge of whole number place Value. The appropriate use of the Commutative property in the application of the Distributive Property. Call out a 2 or 3-digit number and have the students write the expanded form and hold up the simplified numeric expression. Conduct a ten-minute oral Spiral Review of the addition and multiplication basic facts and multiples of 10 while spot checking for proficiency in rewriting/writing whole numbers in expanded form using base ten.

Launch: Display and present the following word problem – The Pre-Winter Back-To-School advertisement at Pretlow's Dollars store features all size back packs at $12.00 each and ink pens at $1.00 each. You plan to purchase five back packs and 5 packs of colored pens. Describe and illustrate two methods or strategies in which you can find the total of your purchases. *5 (10 + 2) times 5(2 +3)*

Sample: **10**   **+**   **2**

5

×   **2**   **3**

5
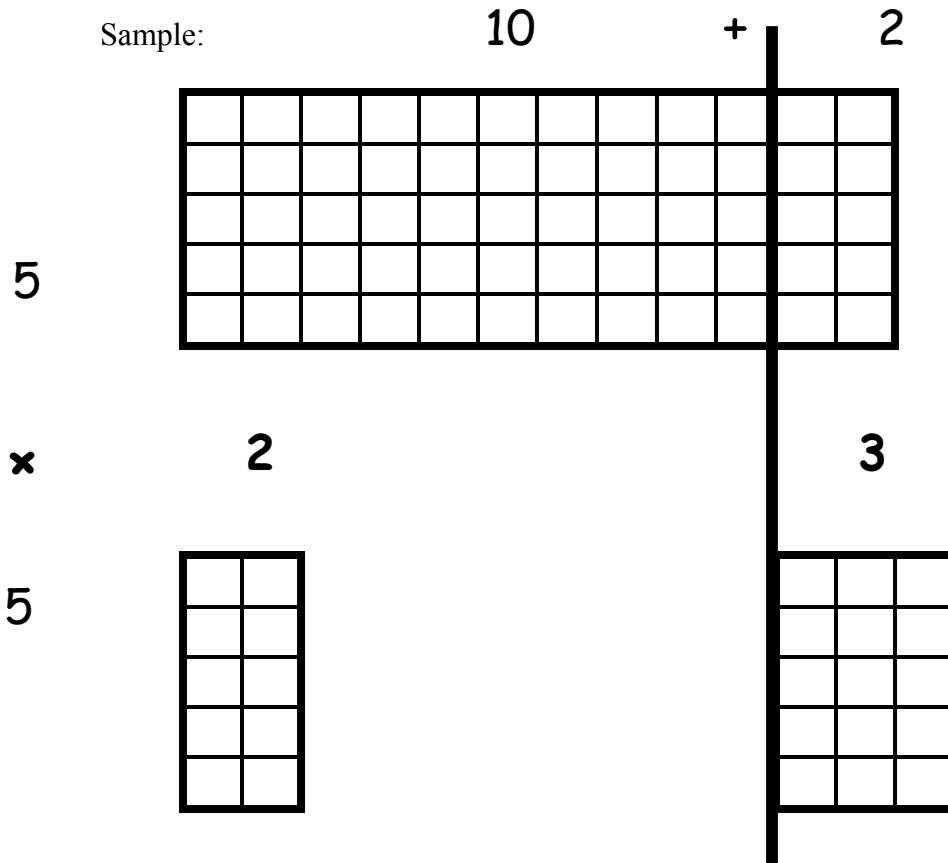
Teacher Facilitation: Teacher models F. O. I. L. Investigation (TR 5). Through a memory game format, the distributive property is extended to include the mental computation of the disaggregate of two factors.

[Follow

instructions written on the F. O. I. L. Investigation (TR 5)

Sample of F.O.I.L. Investigation cards:

12 × 6
(2 + 10) × (2 + 3)

First = a x c   Outer = a x d   Inner = b x c   Last = b x d

Student Application: Teacher forms small groups of threes by using any applicable pre-assessment results to identify students who have demonstrated mastery/proficiency in multiplying using multiples of tens. Student must keep a log recording all incorrect responses. Student copies in Math Journal all incorrectly answered numeric expressions.

Embedded Assessment: Students complete a flow chart to show the process they used to compute the totals, using mental math only.

Re-teaching/Extension: Using the previously recorded missed/incorrect numeric expressions, the teacher conducts small group intensive instruction. Flow charts will be used for re-teaching. Use the Multiple Focus 10, TR 3. Avoid paper and pencil as much as possible.

Summative Assessment: See SR 6

DESCRIPTION:
1) The learners will apply the distributive property to simplify numeric expressions of whole numbers, 0-1000 by generating and illustrating, on graph paper, one story problem reflecting the application of the distributive property in everyday living (such as, finding perimeter of a rectangle shaped bedroom, student spending summer vacation time with two sets of grandparents (maternal and fraternal); amount of fruits and vegetables you eat during a thirty-day period.)

2) The learners will orally demonstrate mental computation of numeric expressions: Use numeric expressions from SR 3 and TR 5.

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EM-TAP

M U L T I P L Y

A d d

Expand	Represent
	Rule:
Multiply	**Add Products**

0	1	2
3	4	5
6	7	8
9	+	-
	=	

Multiple Focus 10

Create the given examples on 3 x 5 single-colored index cards.

820	20 x 41	500	10 x 50
1,650	30 x 55	400	100 x 4
630	7 x 90	1,750	35 x 50
360	18 x 20	2,550	30 x 85

Sample: One side of the card holds a product **or** factors, the other side is labeled with the game words **Multiple Focus 10**.

20 x 41	Multiple Focus 10

MULTIPLICAND BREAKDOWN

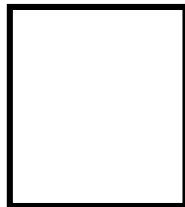| FACTOR	Hundreds	Tens	Ones
X Factor			

F.O.I.L. Investigation

Make letter cards shown below on Index Cards. Scramble and place face down.
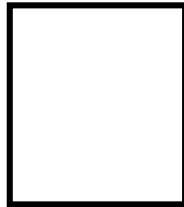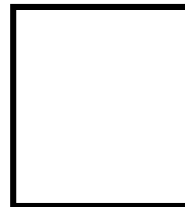
F = a x c   O = a x d   I = b x c   L = b x d

Make the following numeric expression cards, each on 4 x 6 tag board.

12 x 6 (2+10) x (3+3)	18 x 24 (10+8) x (20+4)	84 x 72 (80+4) x (70+2)
8 x 7 (3+5) x (2+5)	32 x 45 (30+2) x (40+5)	6 x 15 (3+3) x (10+5)

Apply the distributive property to simplify numeric expressions of whole numbers.

Everyday you have purchased one slice of pizza and one liter of soda. One slice of pizza is $3.00 and one liter of your favorite soda costs $2.00. Your mother has repeatedly warned that junk foods rob you of good health and your savings. So, you decided to keep a 14-day log of your spending.

Write the numeric expression that describes your problem.

_____

Write two ways to simplify the numeric expression.

1. _____

 2. _____

Describe and model both procedures on the work papers provided.